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Instabilities and replica-symmetry breaking in the four-state clock spin glass

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Abstract. The infinite-range four-state clock spin glass, in the presence of an arbitrary magnetic field, is investigated by the replica method. The appropriate replica-symmetry breaking scheme is discussed for this model. It is shown that two distinct Almeida–Thouless lines are possible, depending on the direction of the applied field. The full phase diagram of the model, showing its instability regions, is presented and discussed.

1. Introduction

The issue of spin glasses has become one of the most intriguing problems in the physics of disordered systems in recent years [1, 2]. Its mean-field theory, in which the prototype is based on the solution of the infinite-range Ising spin glass, i.e. the Sherrington–Kirkpatrick (SK) model [3], has posed many unusual properties, which were, at the beginning, peculiar to this model. One of the most striking novelties was the Almeida–Thouless (AT) line [4], signalling a phase transition in the presence of an external magnetic field. Below the AT line, the correct description of the system is given in terms of an infinite number of order parameters, i.e. an order parameter function [5], associated with many local free-energy minima.

The outstanding question of the moment concerns the applicability of this theory for the description of real spin glasses. Based on domain-wall renormalization-group arguments for spin glasses [6, 7], Fisher and Huse [8] proposed a model, known as the droplet model, the main conclusions of which are contrary to those of the SK model. Essentially, they claim that the AT line is an artefact of mean-field theory and that, in any *finite* dimension, the spin–glass phase should be described in terms of a single thermodynamic state (together with its corresponding time-reversed one). The droplet model has been seriously criticized [9, 10], and the latest numerical simulations suggest its failure for $d = 4$ [11, 12] but are not conclusive for $d = 3$ [12, 13]. In addition, recent Monte Carlo computations in high but finite dimensions [14] exhibit the same SK features, and it has been argued that standard renormalization-group approaches are not appropriate for spin glasses [15], which seem to present new universality rules concerning critical exponents [16, 17]. Many experimental realizations [1] claim to have observed a line corresponding to strong irreversibility effects,

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which could well be identified with the AT line. So, it may be that some of the features predicted by mean-field theory could be present in real systems. Besides that, the formalism developed for the SK model has been applied in many other problems, e.g. optimization and neural networks. Hence, the study of infinite-range spin–glass models deserves much attention for their great relevance.

Although the mean-field theory of the Ising spin glass is considered nowadays to be well understood through the solution of the SK model, its generalizations to more complex spin variables has led to many novel questions. For continuous spins (m-vector spin glasses ($m \geqslant 2$)), a different transition line in a field is obtained [18, 19], whereas in what concerns replica-symmetry breaking (RSB) [5], no qualitative changes are noticed [20, 21]. For discrete spin variables, one may observe qualitative changes in both RSB [22–24] and transitions in a field [25].

In this paper we investigate the four-state clock spin glass [26] in the presence of an arbitrary magnetic field. We discuss what should be the appropriate recipe for implementation of the Parisi scheme for a $2n$-replica Ising spin glass with different magnetic fields on each n-replica group. We also show that, depending on the direction of the applied field, two AT lines are possible, each associated with an Ising-model instability. In section 2 we introduce the model and discuss its main properties. In section 3 we present the phase diagram of the model in the presence of an arbitrary magnetic field, as well as the appropriate RSB scheme. Finally, in section 4 we present our conclusions.

2. The model

We shall define the four-state clock spin glass [26] in three equivalent ways (hereafter referred to as representations I, II and III); each definition will turn out to be more appropriate for the description of certain properties. We first introduce the model in terms of its XY Cartesian components (representation I), defined through the Hamiltonian

$$\mathcal{H} = -\sum_{(ij)} J_{ij}\left(S_{ix}S_{jx} + S_{iy}S_{jy}\right) - h_x \sum_i S_{ix} - h_y \sum_i S_{iy} \tag{2.1}$$

where $\sum_{(ij)}$ denotes a sum over all distinct pairs of spins ($i = 1, 2, \ldots, N$) and $\{J_{ij}\}$ are quenched random couplings distributed according to,

$$P(J_{ij}) = (N/2\pi J^2)^{1/2} \exp(-NJ_{ij}^2/2J^2). \tag{2.2}$$

The spin components can only assume the discrete values

$$S_{ix} = \cos \tfrac{1}{2}\pi k_i \qquad S_{iy} = \sin \tfrac{1}{2}\pi k_i \qquad (k_i = 0, 1, 2, 3) \tag{2.3}$$

and the external magnetic field, $\boldsymbol{h} = (h_x, h_y)$, is allowed, in principle, to point in any direction in the XY plane.

An equivalent definition of the four-state clock spin glass is given by introducing two Ising variables (representation II) at each site ($\tau_i, \sigma_i = \pm 1$), through the change of variables

$$S_{ix} = \tfrac{1}{2}(\tau_i + \sigma_i) \qquad S_{iy} = \tfrac{1}{2}(\tau_i - \sigma_i) \tag{2.4}$$

in terms of which the Hamiltonian in (2.1) may be written as

$$\mathcal{H} = -\sum_{(ij)} \frac{J_{ij}}{2}(\tau_i\tau_j + \sigma_i\sigma_j) - h_\tau \sum_i \tau_i - h_\sigma \sum_i \sigma_i \tag{2.5}$$

with

$$h_\tau = \tfrac{1}{2}(h_x + h_y) \qquad h_\sigma = \tfrac{1}{2}(h_x - h_y). \tag{2.6}$$

Within this representation, the model is described in terms of two *independent* Ising systems, each with coupling constants rescaled by a factor of $\frac{1}{2}$ (with respect to those of equation (2.1)), and in the presence of its own magnetic field. Hence, the partition function of the whole system, for a given bond realization, $Z\{J_{ij}\}$ is factorizable:

$$Z\{J_{ij}\} = Z_\tau\{J_{ij}\}Z_\sigma\{J_{ij}\} \tag{2.7}$$

such that the average over the disorder (which is taken on $\ln Z\{J_{ij}\}$, in the quenched case), leads to a free energy per spin,

$$f = f_\tau + f_\sigma. \tag{2.8}$$

This additivity property, as stated above, should be preserved by any formalism used to deal with this model.

As usual, in order to perform the average over the disorder, $[\ ]_J$, one makes use of the replica trick [1], by means of which the free energy per spin becomes

$$-\beta f = \lim_{N\to\infty} \frac{1}{N}[\ln Z]_J = \lim_{N\to\infty}\lim_{n\to 0} \frac{1}{Nn}([Z^n]_J - 1) \tag{2.9}$$

where Z^n is the partition function of n independent copies (or replicas) of the system.

Assuming that the limits in (2.9) can be freely interchanged, one may use the steepest descent method to evaluate $[Z^n]_J$ and get

$$\beta f = \lim_{n\to 0} \frac{1}{n}\min[g] \tag{2.10}$$

where, in representation I,

$$g(R^\alpha, \{Q^{\alpha\beta}_{\mu\nu}\}) = -\frac{n}{8}(\beta J)^2 + \frac{(\beta J)^2}{2}\sum_\alpha (R^\alpha)^2 + \frac{(\beta J)^2}{2}\sum_{(\alpha\beta)}\sum_{\mu\nu}(Q^{\alpha\beta}_{\mu\nu})^2 - \ln \mathrm{Tr}_\alpha \exp\{\mathcal{H}_{\mathrm{eff}}\} \tag{2.11a}$$

$$\mathcal{H}_{\mathrm{eff}} = (\beta J)^2 \sum_\alpha R^\alpha[(S^\alpha_x)^2 - \tfrac{1}{2}] + (\beta J)^2 \sum_{(\alpha\beta)}\sum_{\mu\nu} Q^{\alpha\beta}_{\mu\nu} S^\alpha_\mu S^\beta_\nu + \beta h_x \sum_\alpha S^\alpha_x + \beta h_y \sum_\alpha S^\alpha_y. \tag{2.11b}$$

In the equations above, α and β are replica labels ($\alpha, \beta = 1, 2, \ldots, n$), μ and ν denote Cartesian components ($\mu, \nu = x, y$) and $\sum_{(\alpha\beta)}$ stand for sums over distinct pairs of replicas. The free-energy extrema give us the equations of state

$$R^\alpha = \langle (S^\alpha_x)^2 \rangle - \tfrac{1}{2} \tag{2.12a}$$

$$Q^{\alpha\beta}_{\mu\nu} = \langle S^\alpha_\mu S^\beta_\nu \rangle \qquad (\alpha \neq \beta). \tag{2.12b}$$

In a similar way, in representation II one obtains

$$g(q_\tau^{\alpha\beta}, q_\sigma^{\alpha\beta}, t^{\alpha\beta}) = -\frac{n}{8}(\beta J)^2 + \frac{(\beta J)^2}{8}\sum_{(\alpha\beta)}(q_\tau^{\alpha\beta})^2 + \frac{(\beta J)^2}{8}\sum_{(\alpha\beta)}(q_\sigma^{\alpha\beta})^2 + \frac{(\beta J)^2}{8}\sum_{\alpha\beta}(t^{\alpha\beta})^2$$

$$- \ln \mathrm{Tr}_\alpha \exp\{\mathcal{H}_{\mathrm{eff}}\} \tag{2.13a}$$

$$\mathcal{H}_{\mathrm{eff}} = \frac{(\beta J)^2}{4}\sum_{(\alpha\beta)} q_\tau^{\alpha\beta}\tau^\alpha\tau^\beta + \frac{(\beta J)^2}{4}\sum_{(\alpha\beta)} q_\sigma^{\alpha\beta}\sigma^\alpha\sigma^\beta + \frac{(\beta J)^2}{4}\sum_{\alpha\beta} t^{\alpha\beta}\tau^\alpha\sigma^\beta + \beta h_\tau \sum_\alpha \tau^\alpha$$

$$+ \beta h_\sigma \sum_\alpha \sigma^\alpha \tag{2.13b}$$

where the summations over $t^{\alpha\beta}$ are now totally unrestricted and

$$q_\tau^{\alpha\beta} = \langle\tau^\alpha\tau^\beta\rangle \qquad q_\sigma^{\alpha\beta} = \langle\sigma^\alpha\sigma^\beta\rangle \qquad (\alpha \neq \beta) \tag{2.14a}$$

$$t^{\alpha\beta} = \langle\tau^\alpha\sigma^\beta\rangle \qquad (\text{any } \alpha, \beta). \tag{2.14b}$$

The parameters $t^{\alpha\beta}$, which measure the correlations in replica space between two originally-independent Ising models, are generated in the averaging process [27, 28], due to the fact that *both* models are subject to the *same disorder*. One should keep in mind that, after taking the appropriate $n \to 0$ limit, the additivity property (2.8) must be restored. For this to happen, one should have either one of the following possibilities:

(i) the t-dependence is completely removed after the $n \to 0$ limit is performed;

(ii) the parameters $t^{\alpha\beta}$ factorize as

$$t^{\alpha\beta} = \langle\tau^\alpha\sigma^\beta\rangle = \langle\tau^\alpha\rangle\langle\sigma^\beta\rangle = m_\tau^\alpha m_\sigma^\beta \qquad (\text{any } \alpha, \beta) \tag{2.15}$$

where m_τ^α and m_σ^β are magnetization parameters for the τ and σ systems, respectively. In this case, one can prove the aditivity property (2.8).

At this point, one may choose particular parametrizations for the matrix elements (2.12) (or (2.14)). The simplest of them is the replica symmetry (RS) ansatz [3] which, for representation II, is given by

$$q_\tau^{\alpha\beta} = q_\tau q_\sigma^{\alpha\beta} = q_\sigma \qquad (\text{all } \alpha \neq \beta) \tag{2.16a}$$

$$t^{\alpha\beta} = t \qquad (\text{all } \alpha, \beta). \tag{2.16b}$$

Within this choice, one may show that possibility (i) above is satisfied, whereas (ii) is not, and therefore requirement (2.8) is fulfilled (see appendix). However, it is a well-known fact that such parametrization leads to instabilities at low temperatures [4], and one must look for other types of solutions.

Now, we introduce another representation for the four-state clock spin glass (representation III). Let us consider an SK model within a $2n$-replica space, such that in each n-replica group a different magnetic field is applied. Defining

$$\xi^b = \begin{cases} \tau^\alpha & (b = 1, \ldots, n) \\ \sigma^\alpha & (b = n+1, \ldots, 2n) \end{cases}$$

one has a $(2n \times 2n)$ matrix (Γ), with elements, $\Gamma^{ab} = \langle \xi^a \xi^b \rangle$, given by

$$\Gamma = \begin{pmatrix} q_\tau^{\alpha\beta} & t^{\alpha\beta} \\ t^{\alpha\beta} & q_\sigma^{\alpha\beta} \end{pmatrix}.$$

The free-energy functional in (2.13) becomes

$$g(\Gamma^{ab}) = -\frac{n}{8}(\beta J)^2 + \frac{(\beta J)^2}{8} \sum_{(ab)} (\Gamma^{ab})^2 - \ln \mathrm{Tr}_\xi \exp\{\mathcal{H}_{\mathrm{eff}}\} \tag{2.17a}$$

$$\mathcal{H}_{\mathrm{eff}} = \frac{(\beta J)^2}{4} \sum_{(ab)} \Gamma^{ab} \xi^a \xi^b + \beta h_\tau \sum_{b=1}^{n} \xi^b + \beta h_\sigma \sum_{b=n+1}^{2n} \xi^b. \tag{2.17b}$$

Within this representation, such a model has been the subject of much interest and some controversy in the literature [1, 28–31], in particular as to what is the most appropriate way to carry out the RSB process. This point will be discussed in the next section, together with the instabilities of the RS solutions (2.16).

3. Replica-symmetry breaking

In the case $h_\tau = h_\sigma$ it is clear that the zeroth step in the RSB process corresponds to a matrix Γ with diagonal elements $\Gamma^{bb} = 0$ and equal off-diagonal ones, i.e.

$$q_\tau = q_\sigma = t. \tag{3.1}$$

The effect of a small difference in the fields, $h_\tau \neq h_\sigma$, is to break the permutation symmetry of the $2n$ replicas. Basically, two ways of performing the RSB for the $2n \times 2n$ matrix $\mathbf{\Gamma}$ have been discussed in the literature [1], as we mention below.

(i) Split the matrix into distinct $n \times n$ blocks at the lowest level, such that

$$q_\tau^{\alpha\beta} = q_\tau(1 - \delta_{\alpha\beta}) \qquad q_\sigma^{\alpha\beta} = q_\sigma(1 - \delta_{\alpha\beta}) \qquad t^{\alpha\beta} = t \tag{3.2}$$

where $\delta_{\alpha\beta}$ is Kronecker's delta. This can be seen as a particular choice for the first RSB step in the case $h_\tau = h_\sigma$. Within this procedure, a small difference in the fields corresponds to raising the order in the RSB, defining the start of the process. At this level, one should notice that the free energy is independent of the parameter t, as discussed in the previous section. Now, Parisi's recipe is implemented for the diagonal blocks, whereas the off-diagonal ones are left untouched.

(ii) App.. 'arisi's RSB recipe for each $n \times n$ block separately [1, 31], setting the diagonal elements $q_\tau^{\alpha\alpha} = q_\sigma^{\alpha\alpha} = 0$ and $t^{\alpha\alpha} = \Gamma(1)$. The resulting free energy will depend on $t^{\alpha\alpha} = \Gamma(1)$ [1], which is given in the appendix (equation (A.8)).

We argue that only procedure (i) is physically acceptable, since the resulting free energy will present no t-dependence, preserving the additivity property (2.8). Procedure (ii) will not satisfy (2.8) since the parameters $t^{\alpha\alpha}$ are not factorizable as products of magnetizations (see the appendix).

Within procedure (i), one ends up with

$$f[q_\tau(x), q_\sigma(x)] = f_\tau[q_\tau(x)] + f_\sigma[q_\sigma(x)] \tag{3.3}$$

where $f_\tau[q_\tau(x)]$ ($f_\sigma[q_\sigma(x)]$) is the corresponding Ising free-energy functional [32, 33] (with the rescaling $J \to J/2$) of the $\tau(\sigma)$ system.

Let us now consider the instabilities of the RS solution, in the general case of an arbitrary magnetic field $\boldsymbol{h} = (h_x, h_y)$. Due to the result (3.3) it is clear that one will obtain distinct instability regions, each associated with the RSB of an Ising spin glass in the presence of its respective field, given in (2.6). Such instabilities will be AT-like and only in the particular case of a field aligned along one of the Cartesian axis, $\boldsymbol{h} = (h_x, 0)$ for which $h_\tau = h_\sigma$, or $\boldsymbol{h} = (0, h_y)$ where $h_\tau = -h_\sigma$, does one obtain a single AT line. The general picture consists of two independent surfaces which split the (h_x, h_y, T) space into regions, as shown in figure 1 (the AT line has been reproduced numerically many times before [1], so we just present its usual form in this problem), where both systems (τ and σ) are in RS; one of the systems is in RS and for the other, RS should be broken; both systems should be treated within the RSB. In figure 1 we show the intersections of these surfaces with planes of the type $h_x = ch_y$ ($c \equiv$ constant) in two cases, i.e. $c > 1 (h_\tau > h_\sigma > 0)$ and $c = 1$ $(h_\tau > h_\sigma = 0)$. In the former case ($c > 1$) one finds two AT lines. For a fixed value of the magnetic field, at high temperatures, both Ising systems are stable within RS. By lowering the temperature, one meets the first AT line ($T = T_\tau^{\mathrm{AT}}(h_\tau)$), associated with RSB in the τ system only, and by further decreasing T one meets the second AT line ($T = T_\sigma^{\mathrm{AT}}(h_\sigma)$), associated with the σ system. The region in between the two lines ($T_\sigma^{\mathrm{AT}}(h_\sigma) < T < T_\tau^{\mathrm{AT}}(h_\tau)$), is characterized by RSB in the τ system and RS in the σ system, whereas for $T < T_\sigma^{\mathrm{AT}}(h_\sigma)$, both should be treated within the RSB. In the latter case ($c = 1$) the two lines are superposed, signalling a crossover, in the sense that for $c > 1$ the τ-line appears at a higher temperature, $T_\tau^{\mathrm{AT}}(h_\tau) > T_\sigma^{\mathrm{AT}}(h_\sigma)$, whereas for $c < 1$, the lines are switched, i.e. $T_\tau^{\mathrm{AT}}(h_\tau) < T_\sigma^{\mathrm{AT}}(h_\sigma)$.

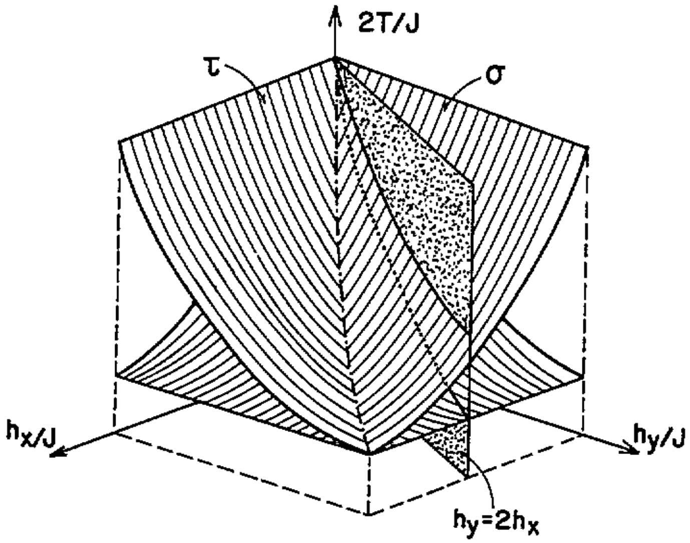

Figure 1. Schematic plot showing the two independent instability surfaces in the (h_x, h_y, T) space, inside a cube with side of unitary length. Intersections of these surfaces with the plane $h_y = 2h_x$ are represented, showing two AT lines: the higher-temperature one corresponds to an instability in the σ system (full curve), whereas the lower-temperature one is associated with the τ system (dotted curve).

4. Conclusion

We have studied the infinite-range four-state clock spin glass in the presence of an arbitrary-direction magnetic field in the XY plane. Within one of its representations, i.e. two independent Ising systems, we have discussed the RSB scheme for this model. From the two usually presented RSB procedures, we have argued that only one of them is physically acceptable, as it preserves the additivity of the free energies of the independent Ising systems. The instabilities of the RS solution are of the AT type; two surfaces, each associated with an SK model, are presented, separating regions where both, only one or none, of the systems are stable.

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Appendix

In this appendix, we treat the four-state clock spin glass in the RS ansatz (equations (2.16)). Within this approximation, the free energy in (2.10) becomes

$$\beta f = -\frac{(\beta J)^2}{8} + \frac{(\beta J)^2}{8}(q_\tau + q_\sigma) - \frac{(\beta J)^2}{16}[(q_\tau)^2 + (q_\sigma)^2] - \int D[x] \ln(2\cosh\phi) - \int D[x] \ln(2\cosh\psi) \tag{A.1}$$

where the t-dependence outside the integrals has disappeared in the $n \to 0$ limit. In the equation above,

$$\phi = \tfrac{1}{2}\beta J(q_\tau - t)^{1/2}x + \tfrac{1}{2}\beta J t^{1/2} z + \beta h_\tau \tag{A.2a}$$

$$\psi = \tfrac{1}{2}\beta J(q_\sigma - t)^{1/2}y + \tfrac{1}{2}\beta J t^{1/2} z + \beta h_\sigma \tag{A.2b}$$

and

$$\int D[x]\cdots \equiv \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{\sqrt{2\pi}}\frac{\mathrm{d}y}{\sqrt{2\pi}}\frac{\mathrm{d}z}{\sqrt{2\pi}} \exp\left[-\left(\frac{x^2}{2}+\frac{y^2}{2}+\frac{z^2}{2}\right)\right]\cdots. \tag{A.3}$$

The two multiple integrals appearing in (A.1) may be transformed into single integrals and the t-dependence removed, by the following respective changes in variable:

$$q_\tau^{1/2} u = (q_\tau - t)^{1/2}x + t^{1/2}z \tag{A.4a}$$

$$q_\tau^{1/2} \tilde{u} = t^{1/2}x - (q_\tau - t)^{1/2}z \tag{A.4a}$$

$$q_\sigma^{1/2} v = (q_\sigma - t)^{1/2}y + t^{1/2}z \tag{A.5a}$$

$$q_\sigma^{1/2} \tilde{v} = t^{1/2}y - (q_\sigma - t)^{1/2}z. \tag{A.5b}$$

Using this, the free energy becomes

$$f = f_\tau + f_\sigma \tag{A.6}$$

where

$$\beta f_\omega = -\frac{(\beta J)^2}{16}(1-q_\omega)^2 - \int_{-\infty}^{\infty}\frac{\mathrm{d}s}{\sqrt{2\pi}}\exp\left(-\frac{s^2}{2}\right)\ln\left[2\cosh\left(\frac{\beta J}{2}q_\omega^{1/2}s+\beta h_\omega\right)\right] \tag{A.7a}$$

$$q_\omega = \int_{-\infty}^{\infty}\frac{\mathrm{d}s}{\sqrt{2\pi}}\exp\left(-\frac{s^2}{2}\right)\tanh^2\left(\frac{\beta J}{2}q_\omega^{1/2}s+\beta h_\omega\right) \tag{A.7b}$$

with $\omega = \tau(s = u), \sigma(s = v)$, representing the two SK models, each of them with rescaled widths $J/2$.

One may also see that the parameter t in RS is given by

$$t = \int D[x]\tanh\phi\tanh\psi \tag{A.8}$$

which may not be written in the form (2.15), i.e. $t \neq m_\tau m_\sigma$.

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